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THE HEAVY HIGGS RESONANCE

G. Valencia

Fermi National Accelerator Laboratory
P. O. Box 500, Batavia, IL 60510

S. Willenbrock

Physics Department
Brookhaven National Laboratory, Upton, NY 11973 ⋆
and
Fermi National Accelerator Laboratory
P. O. Box 500, Batavia, IL 60510

ABSTRACT

We study the use of the standard Higgs boson, with $m_H > 700$ GeV, as a model for a resonance in longitudinal-vector-boson scattering. We focus on the constraint placed upon the modulus of the amplitude by unitarity. We show that it is better to use the energy-dependent width, $\Gamma(s) = \Gamma_H \times (s/m_H^2)^2$, in the Higgs-boson propagator, rather than the usual approximation, $\Gamma(s) = \Gamma_H$. The s-channel approximation, the effective-W approximation, and the full electroweak $qq \to qqVV$ process are discussed.

⋆ Permanent address

The Higgs boson is a particle associated with the mechanism which breaks the electroweak symmetry in the standard model. The Higgs-boson mass is a free parameter of the model. However, given its mass, and the masses of all particles to which it couples, its properties are completely determined by the electroweak theory. The CERN Large Hadron Collider (LHC) and the US Superconducting Super Collider (SSC) are being designed to search for the Higgs boson from a mass of about 80 GeV (the upper limit accessible to the CERN LEP II e^+e^- collider) up to nearly 1 TeV.

Although the Higgs boson's properties are governed by the electroweak interaction, its couplings are not necessarily of ordinary electroweak strength, $\mathcal{O}(g^2)$. In particular, the Higgs boson couples to longitudinal weak vector bosons with strength $\mathcal{O}(g^2 m_H^2/M_W^2)$; thus, if $m_H^2 \gg M_W^2$, its interaction with these particles is enhanced [1,2].[#1] We will concern ourselves only with terms of enhanced electroweak strength in this paper.

If the Higgs boson is much heavier than M_W, it couples to longitudinal vector bosons so strongly that perturbation theory becomes unreliable. Theoretical considerations, based on perturbative unitarity, suggest that this occurs at $m_H \approx 700$ GeV [1-4] or less [5]. A separate argument, based on the triviality of $\lambda\phi^4$ theory, suggests that the standard Higgs model ceases to exist at about 650 GeV, in the sense that effects =associated with physics beyond the Higgs model must enter at energies not much greater than the Higgs-boson mass [6].[#2] Neither of these arguments preclude the possibility that a scalar resonance occurs in longitudinal-vector-boson scattering above 700 GeV, but should such a resonance be discovered,

#1 Below the Higgs resonance, the enhancement factor is s/M_W^2; above, it is m_H^2/M_W^2. This distinction is unimportant in the resonance region.

#2 In Ref. 7 it is suggested that this bound may be relaxed to about 850 GeV.

its interpretation as the standard Higgs boson would not be evident.

Many studies have been undertaken to establish the capability of the LHC/SSC to discover a Higgs boson with a mass in excess of 700 GeV [8-24]. In view of the above arguments, it is justifiable to question the validity of such studies. However, one may adopt a purely phenomenological point of view, and regard the Higgs boson with a mass greater than 700 GeV as a *model* for a resonance in longitudinal-vector-boson scattering. As long as one calculates at tree level, the difficulties associated with the breakdown of perturbation theory and the triviality of the theory are seemingly avoided.

In this paper we study the use of the standard Higgs boson, with $m_H > 700$ GeV, as a model for a resonance in longitudinal-vector-boson scattering. We pay particular attention to the constraint placed upon the modulus of the amplitude by unitarity. Unitarity imposes a *nonperturbative* upper bound on the modulus of the J^{th} partial-wave amplitude, thereby bounding the cross section in that partial wave. We will show that almost all perturbative calculations violate this bound at the peak of the Higgs resonance, and we will quantify to what extent it is violated. Large violations of the bound are unacceptable in that they lead to cross sections which cannot be physically realized. Thus the difficulties associated with strong coupling manifest themselves already at tree level.

Several approaches to the calculation of longitudinal-vector-boson scattering have been employed. The complete set of electroweak diagrams for $qq \to qqVV$, where $VV = W^+W^-, ZZ$, has been calculated by several groups [8-11]. The terms of enhanced electroweak strength can be obtained via the effective-W approximation [25], which was used to calculate vector-boson scattering before the availability of the complete $qq \to qqVV$ calculation [12,13]. It is still commonly employed in shower Monte Carlo programs,#3 for which the complete calculation is too time consuming, and in calculations of longitudinal-vector-boson scattering in models other than the standard Higgs model [12,14]. The so-called "s-channel" approximation, in which only the diagram $qq \to qqH^* \to qqVV$ is kept (the asterisk denotes a virtual Higgs boson), is also sometimes used, both in shower Monte Carlo#3 and analytic calculations [22-24].

Unitarity of the S matrix is simplest when applied to elastic scattering amplitudes of definite angular momentum and "custodial" isospin, a_J^I. As is well known, unitarity implies the bound#4

$$|a_J^I| \leq 1 \ . \tag{1}$$

In the elastic region, below the threshold for multi-particle production, unitarity implies the stronger condition

$$\mathrm{Im}\, a_J^I = |a_J^I|^2. \tag{2}$$

Ordinarily, amplitudes calculated perturbatively do not satisfy the conditions of unitarity exactly, but only up to the order at which they are calculated. This is entirely acceptable within the context of weak-coupling perturbation theory. However, when the coupling becomes strong, the breakdown of perturbation theory is often accompanied by large violations of the unitarity bound, Eq. (1) [1,2].

We begin by considering the Higgs-boson propagator near the Higgs-boson resonance. As usual, one performs a Dyson summation of the one-particle-irreducible

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#3 Heavy Higgs boson studies using shower Monte Carlo programs may be found in Refs. 14-17 and in the proceedings of various workshops [18-21]. For a summary of which approximations are currently available in PYTHIA, ISAJET, and HERWIG, see Ref. 26.

#4 We neglect the vector-boson masses throughout.

Higgs-boson self-energy graphs to obtain

$$D(s) = \frac{i}{s - m_0^2 + \Pi(s)}, \tag{3}$$

where m_0 is the bare mass. In the resonance region, the propagator may be approximated by [27]

$$D(s) = \frac{i}{s - m_R^2 + i \,\mathrm{Im}\,\Pi(s)}, \tag{4}$$

where m_R is a renormalized mass parameter. The imaginary part of $\Pi(s)$ is related to the Higgs-boson width via unitarity. Since we are only interested in terms of enhanced electroweak strength, we need only insert longitudinal-vector-boson intermediate states into the unitarity relation (in unitary gauge). We find

$$\mathrm{Im}\,\Pi(s) = \frac{1}{16\pi} \, |V(s)|^2, \tag{5}$$

where $V(s)$ is the HV_LV_L three-point function. Since $V(s) \sim s$ at tree level, due to the longitudinal polarization vectors, we obtain

$$\mathrm{Im}\,\Pi(s) \approx m_R \Gamma_H \left(\frac{s}{m_R^2}\right)^2 \equiv m_R \Gamma(s), \tag{6}$$

where Γ_H is the Higgs-boson width. Note that $\Gamma(s) \sim s^2$, in contrast with the Z boson, where $\Gamma(s) \sim s$ [27].

The tree-level, $I = 0$, $J = 0$ partial-wave amplitude for $V_LV_L \to V_LV_L$ is

$$a_0^0 = -\frac{m_R\Gamma(s)}{s - m_R^2 + i m_R \Gamma(s)} + b_0^0, \tag{7}$$

where we have written the numerator of the s-channel Higgs-boson exchange term, $(V(s))^2$, in terms of $\Gamma(s)$ via Eqs. (5) and (6). The term b_0^0 is the zeroth partial wave of the nonresonant, "background" graphs,

$$b_0^0 = -\frac{\lambda}{8\pi}\left[1 - \frac{3}{2}\,\frac{s}{m_R^2} - \frac{m_R^2}{s}\ln\left(1 + \frac{s}{m_R^2}\right)\right], \tag{8}$$

and the tree-level width is given by

$$\Gamma_H = \frac{3}{16\pi}\lambda m_R, \tag{9}$$

where

$$\lambda \equiv \frac{1}{8}g^2\frac{m_R^2}{M_W^2} \equiv \frac{G_F m_R^2}{\sqrt{2}} \tag{10}$$

is the enhanced electroweak coupling. Only terms of $\mathcal{O}(\lambda)$ have been maintained in these expressions.

We first study the s-channel approximation, which corresponds to keeping only the first term in Eq. (7). In the resonance region, defined by $|s - m_R^2| \lesssim m_R\Gamma_H$, the denominator of the Higgs-boson propagator is $\mathcal{O}(\lambda)$, so this term is $\mathcal{O}(1)$, while the background term, b_0^0, is $\mathcal{O}(\lambda)$. Thus the s-channel approximation is the leading term in a perturbative expansion of the amplitude in the resonance region [27].

The s-channel approximation satisfies the elastic-unitarity condition, Eq. (2), exactly, provided $\Gamma(s)$ is employed in the denominator. However, most s-channel-approximation calculations have used Γ_H in the Higgs-boson propagator, while effectively using $\Gamma(s)$ in the numerator [15,16,18-21,22,24].[#5] This is acceptable for a narrow resonance [23], but leads to large violations of the unitarity bound,

#5 $\Gamma(s)$ is used in the s-channel approximation in the current version of PYTHIA, which was used in Ref. 17, and in the current version of HERWIG (version 5.1) [28]. However, Γ_H was used in previous versions of PYTHIA and HERWIG. ISAJET does not use the s-channel approximation [26].

Eq. (1), for a heavy Higgs boson. This is demonstrated in Figs. 1 and 2, in which we plot $|a_0^0|^2$ versus energy for $m_R = 800$ and 1000 GeV, respectively. The dotted line corresponds to the s-channel approximation using $\Gamma(s)$ in the denominator, while the dot-dashed line is generated using Γ_H there. For $m_R = 800$ GeV, the latter violates the unitarity bound by fifty percent,[#6] while for $m_R = 1$ TeV the bound is violated by as much as a factor of three (at $\sqrt{s} \approx 1400$ GeV). *Thus, in the s-channel approximation, it is better to use $\Gamma(s)$, rather than Γ_H, in the Higgs-boson propagator.*

The s-channel approximation for a heavy Higgs resonance has been criticized on the grounds that it violates the unitarity bound at energies above the resonance [10,20]. We see that this is only true if Γ_H is used in the Higgs-boson propagator; if $\Gamma(s)$ is used, no violation occurs. This is fortuitous, however; outside the resonance region there is no reason to include the width in the propagator, and the s-channel term grows linearly with s. As is well known, the solution is that the nonresonant diagrams cancel the linear growth of the s-channel diagram above the resonance [2,13].

When we add the background term, b_0^0, to the s-channel term in Eq. (7), we find that the elastic unitarity condition, Eq. (2), is no longer satisfied exactly. This is the case at any finite order in the coupling [29]. We show in Figs. 1 and 2 the square of the full tree-level amplitude, Eq. (7), again using $\Gamma(s)$ (solid line) and Γ_H (dashed line) in the Higgs-boson propagator. Although both amplitudes violate the unitarity bound, the violation is worse when Γ_H is used in the propagator; for $m_R = 1$ TeV, the bound is violated by sixty percent. Thus we conclude that *it is*

#6 Actually, it is the square of Eq. (1) that we are discussing. Since the cross section is proportional to $|a|^2$, this is the relevant object.

better to use $\Gamma(s)$*, rather than* Γ_H*, in the Higgs-boson propagator in the calculation of the tree-level amplitude* . However, the existing effective-W approximation [13, 18-21] and full $qq \to qqVV$ calculations [8-11] have routinely used Γ_H in the propagator.[#7]

For $m_R = 1$ TeV, the full tree-level amplitude, with $\Gamma(s)$ in the propagator, begins to grow at $\sqrt{s} \approx 1300$ GeV, eventually violating the unitarity bound at $\sqrt{s} \approx 1500$ GeV. This is because the aforementioned cancellation between the s-channel and nonresonant diagrams is upset by the presence of $\Gamma(s)$ in the propagator. In principle this is not a problem, since outside the resonance region there is no reason to include the width in the propagator. In practice, one can either discard events above 1300 GeV, or multiply $\Gamma(s)$ in the propagator by a function which suppresses it at energies above the resonance.

Several authors have proposed "unitarizing" the tree-level amplitude such that it satisfies elastic unitarity exactly [30,31] or at least satisfies the unitarity bound [12,14]. This is only possible in calculations of longitudinal-vector-boson scattering based on the effective-W approximation, of course. In the case of the full electroweak calculation, $qq \to qqVV$, one cannot easily isolate the $V_LV_L \to V_LV_L$ subprocess. While the effective-W approximation reproduces the terms of enhanced electroweak strength, it cannot reproduce all terms of $\mathcal{O}(g^2)$. Thus the full electroweak calculation contains information which cannot be reproduced by the effective-W approximation [10]. Perhaps the best approach is to calculate the $\mathcal{O}(g^2)$ terms using $qq \to qqVV$ with $m_R \approx 0$, and the $\mathcal{O}(\lambda)$ terms using the

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#7 In Refs. 12 and 14, the Goldstone-boson equivalence theorem was used in conjunction with the effective-W approximation. It is appropriate to use Γ_H in this case, since the coupling of the Higgs boson to the Goldstone bosons is energy independent [29]. However, for $m_R = 1$ TeV, the unitarity bound is violated by seventy percent in the calculation of Ref. 14.

effective-W approximation.

The mass of an unstable particle is conventionally defined as the real part of the pole (in the energy plane) in the particle's propagator. The Higgs-boson mass, m_H, is not equal to the renormalized mass parameter m_R, due (mostly) to the energy-dependence of the width [32]; $m_R = 800$ GeV corresponds to $m_H \approx 750$ GeV, and $m_R = 1$ TeV corresponds to $m_H \approx 900$ GeV.[#8]

The other principal mechanism for producing a heavy Higgs boson at the LHC/SSC is gluon fusion via a top-quark loop [33-35]. The general considerations of this paper also apply there, and the conclusion is again that it is better to use $\Gamma(s)$ in the Higgs-boson propagator, both in the s-channel approximation and in the full calculation of $gg \to V_L V_L$. In both cases, this has generally not been done.[#9] The invariant-mass distributions for ZZ pairs from gluon fusion at the SSC, using $\Gamma(s)$ and Γ_H in the propagator, are shown in Ref. 26 for $m_R = 800$ GeV; the distributions differ considerably.

The top quark also contributes to $\Gamma(s)$. Its contribution is also energy dependent, but linear in s, rather than quadratic. The total $\Gamma(s)$ is the sum of the partial widths.

We have shown that, with regard to unitarity, it is better to use $\Gamma(s) = \Gamma_H \times (s/m_R^2)^2$ in the Higgs-boson propagator, rather than the usual approximation, $\Gamma(s) = \Gamma_H$. We hope that the energy-dependent width will become routinely employed in the Higgs-boson propagator, just as it is in the Z-boson propagator.

#8 When using the Goldstone-boson equivalence theorem, $m_H \approx m_R$ [29].

#9 Exceptions are the current versions of PYTHIA and HERWIG, which use the s-channel approximation with $\Gamma(s)$ [26,28].

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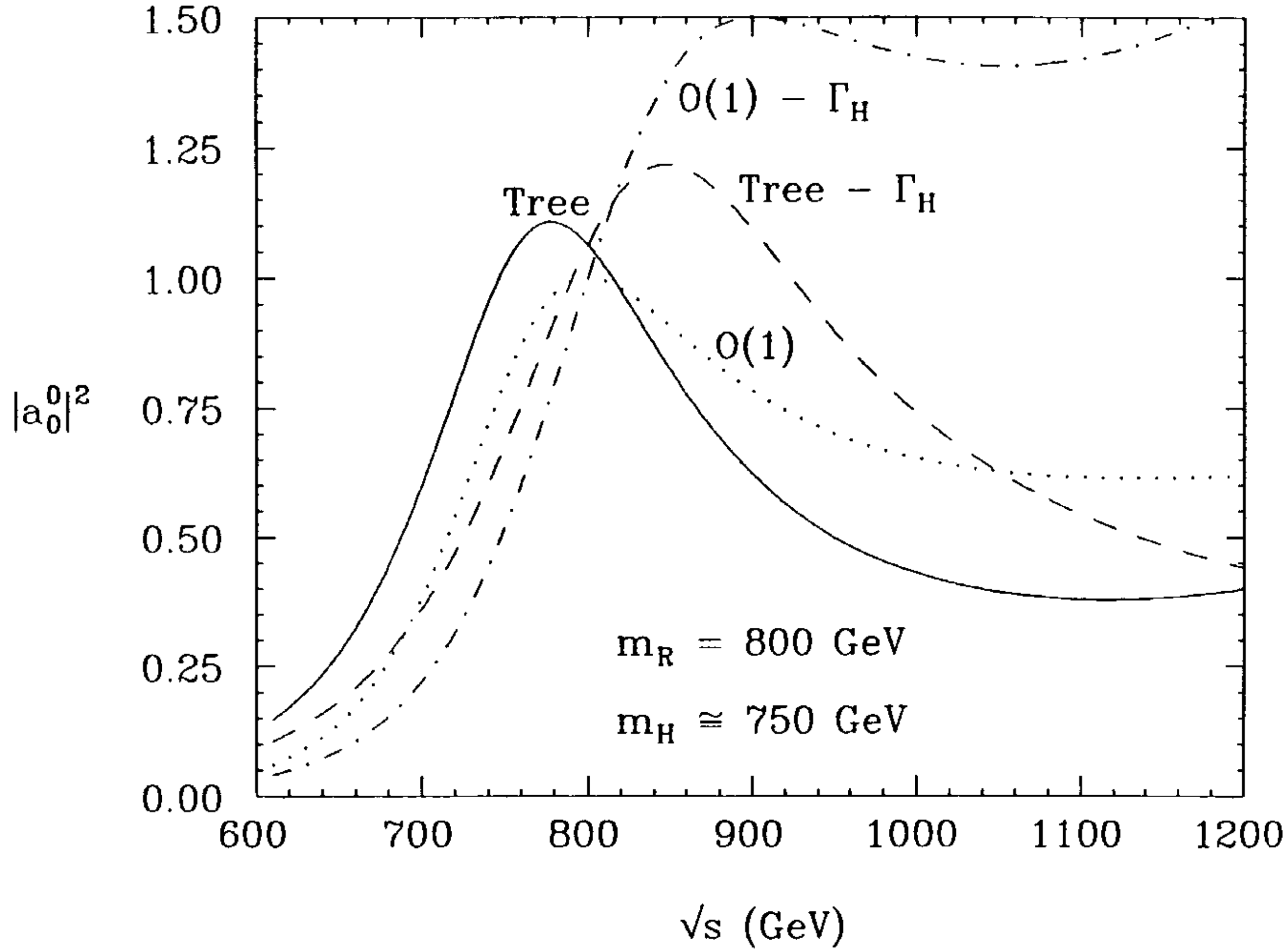
FIGURE CAPTIONS

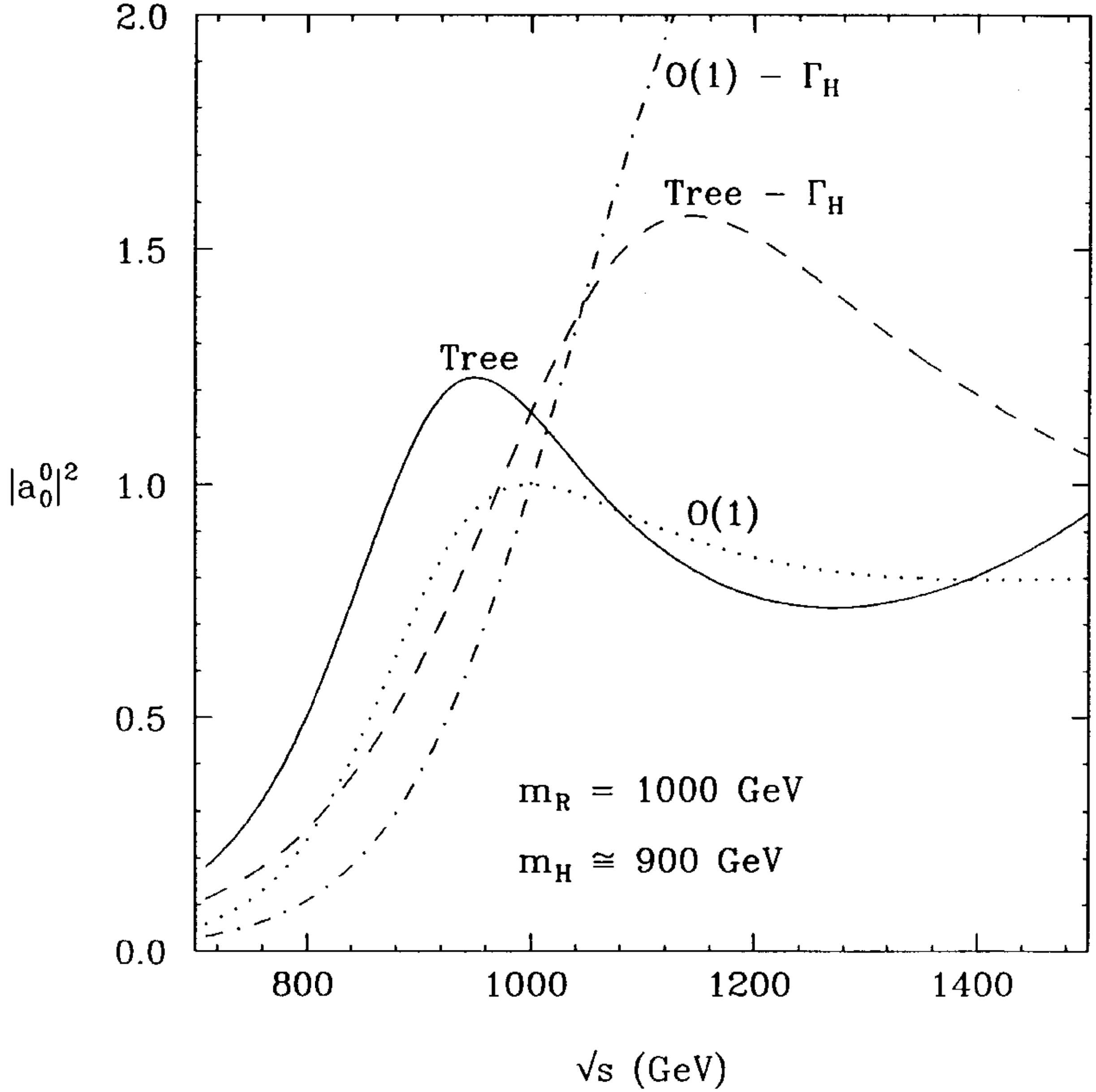
1) The square of the $I = 0$, $J = 0$ partial-wave amplitude for longitudinal-vector-boson scattering versus the total CM energy, for $m_R = 800$ GeV. The curves labeled "$\mathcal{O}(1)$" correspond to the s-channel approximation, while those labeled "Tree" correspond to the full tree-level amplitude. The label "Γ_H" indicates that a constant width is used in the Higgs-boson propagator; otherwise, $\Gamma(s) = \Gamma_H \times (s/m_R^2)^2$ is used. In the latter case, the physical Higgs-boson mass is $m_H \approx 750$ GeV.

2) Same as Fig. 1, but for $m_R = 1000$ GeV. The physical Higgs-boson mass is approximately 900 GeV when $\Gamma(s)$ is used in the Higgs-boson propagator.



$|a_0^0|^2$
1.50
1.25
1.00
0.75
0.50
0.25
0.00
600
700
800
900
1000
1100
1200
$\sqrt{s}$ (GeV)
Tree
O(1) − Γ_H
Tree − Γ_H
O(1)
m_R = 800 GeV
$m_H \cong$ 750 GeV

Fig. 1



2.0
1.5
1.0
0.5
0.0
$|a_0^0|^2$
800
1000
1200
1400
$\sqrt{s}$ (GeV)
O(1) − Γ_H
Tree − Γ_H
Tree
O(1)
m_R = 1000 GeV
$m_H \cong$ 900 GeV

Fig. 2